



## Introduction to wind power models for frequency control studies

Hansen, Anca Daniela

*Publication date:*  
2016

*Document Version*  
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

*Citation (APA):*  
Hansen, A. D. (2016). *Introduction to wind power models for frequency control studies*. DTU Wind Energy E Vol. 104

---

### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# Introduction to wind power models for frequency control studies

DTU Vindenergi  
E Rapport 2016

A. D. Hansen

DTU Wind Energy E-0104

January 2016

**DTU Vindenergi**  
Institut for Vindenergi

---



**Forfatter(e):** A. D. Hansen

**Titel:** Introduction to wind power models for frequency control studies

**Institut:** DTU Wind Energy

**2016**

**Resume (mask. 2000 char.):**

This document covers some basic aspects regarding wind power models, which can be used in power system frequency control studies. Different issues like aerodynamic power, power curve, as well as different wind turbine concepts and their methods to optimize or limit the power extracted from the wind, are thus addressed and briefly discussed.

ISBN 978-87-93278-60-8

**Danmarks Tekniske Universitet**

DTU Vindenergi  
Nils Koppels Allé  
Bygning 403  
2800 Kgs. Lyngby  
Telefon

[www.vindenergi.dtu.dk](http://www.vindenergi.dtu.dk)



## Contents

Introduction.....	5
1. Basic rotor aerodynamics .....	7
Power coefficient.....	8
Aerodynamic power .....	10
Power curve and operational zones.....	10
2. Main wind turbine components.....	11
Generator .....	12
Gearbox .....	13
Power electronic.....	13
3. Wind turbine concepts .....	13
3.1 Fixed speed wind turbines (FSWTs).....	15
3.2 Variable speed wind turbines (VSWTs) .....	20
3.2.1 Introduction on frequency controller of VSWT .....	27
References .....	29

## Introduction

A wind turbine includes several components, which contribute with their specific function in the energy conversion process from wind energy into electrical energy. Figure 1 shows the main components of a wind turbine including the turbine rotor, gearbox, generator, possible power electronics, transformer and finally its connection to the grid.

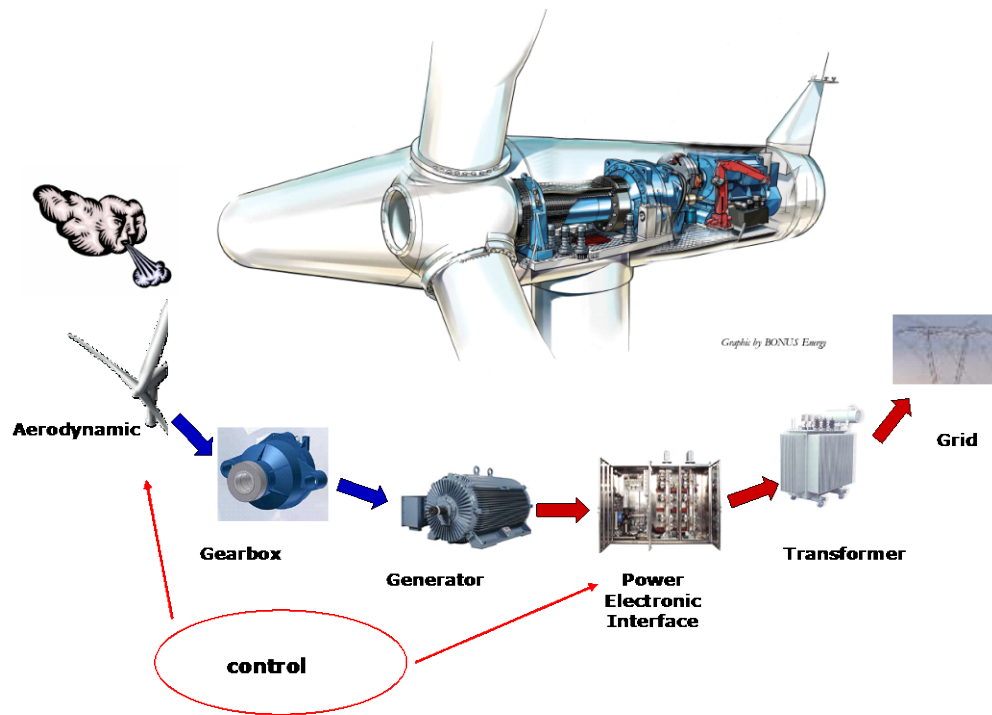


Figure 1: Wind turbine components.

A wind turbine captures the power from the wind by means of its aerodynamic rotor and converts it to kinetic mechanical power. This is further transferred through the gearbox to the generator shaft. The generator converts the mechanical power into electrical power, which is fed into the grid through a power electronic interface and a transformer. A power electronic interface, which is typically present in modern wind turbines, makes it possible to adjust and control the generator frequency and voltage, and thus to enhance wind turbines capability to behave and act as active components in the power system. Wind turbines can be connected at different grid voltage levels due to the presence of step-up transformers, which transform the voltage level existing in the wind turbines to the higher voltage level of local grids. A wind turbine is typically equipped with a control system, necessary to assure a proper operation of the wind turbine under all operational conditions.

Table 1: Nomenclature.

Symbol	Definition	Unit
$U$	wind speed (free stream wind velocity)	[m/s]
$\omega_{wtr}$	rotor speed (rotational speed of the rotor)	[rad/s]
$W$	relative wind speed (resulting wind speed seen by the airfoil)	[m/s]
$v_{tip}$	tip speed (speed of the rotating blade tip)	[m rad/s]
$\lambda$	tip-speed-ratio (ratio between tip speed and wind speed)	[rad]
$\theta$	blade pitch angle (angle between the chord line and the tip speed)	[deg]
$\alpha$	angle of attack (angle between the chord line and the relative wind $W$ )	[deg]
$T$	aerodynamic torque	[kg m <sup>2</sup> /s <sup>2</sup> ]
$P_{wtr}$	aerodynamic power	[W]
$\rho$	air density	[kg/m <sup>3</sup> ]
$A$	rotor swept area	[m <sup>2</sup> ]
$C_p$	power coefficient	[-]
$f_0$	synchronous frequency	[Hz]
$\omega_{e0}$	electrical angular speed	[rad/s]
$\omega_{gen0}$	synchronous generator speed	[rad/s]
$N_{pp}$	number of pole pairs	[-]
$N_{gear}$	gear ratio	[-]
$R$	rotor radius	[m]

## 1. Basic rotor aerodynamics

The aerodynamics of a wind turbine rotor are essential for how much aerodynamic power can be extracted out of the wind by the wind turbine. There are different methods to influence the aerodynamic properties of a wind turbine. In order to understand these, it is necessary to get a brief insight on the aerodynamics of the rotor.

The rotor is the part of the wind turbine that consists both of blades and hub. The rotor starts to rotate due to aerodynamic forces, which appear when the wind passes the blades of the wind turbine. Figure 2 illustrates the front and side view of a wind turbine, as well as a top view of a blade (airfoil).

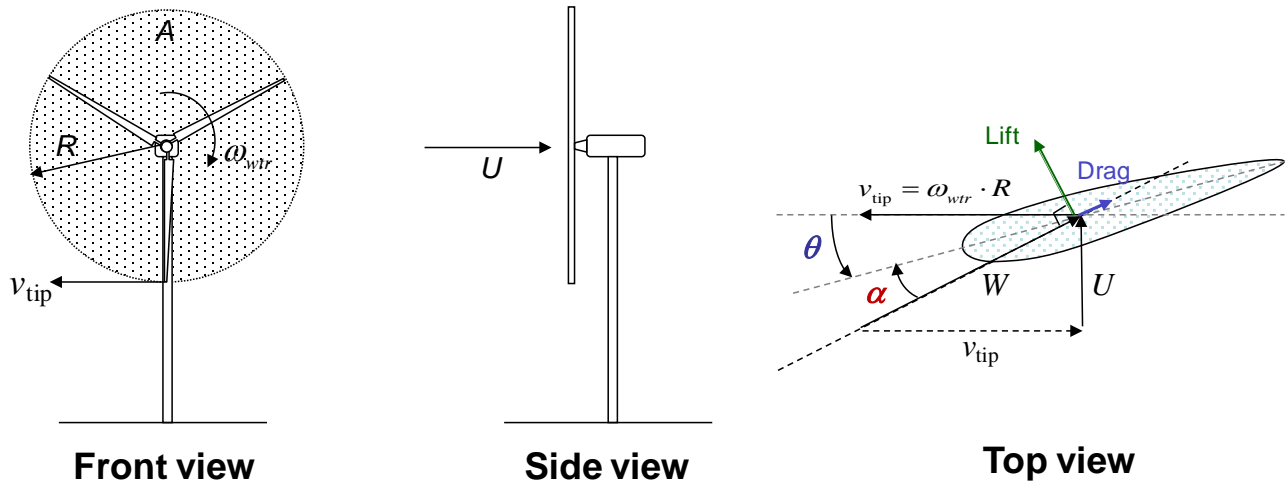


Figure 2: Wind turbine seen from front, side and top.

The notations introduced in Figure 2 are defined in Table 1. However the most important of them are underlined in the following:

- $v_{tip}$  is the tip speed [m/s], i.e. the speed of the rotating blade tip, which can be expressed as:

$$v_{tip} = \omega_{wtr} \cdot R$$

where  $\omega_{wtr}$  the rotational speed of the rotor and  $R$  is the length of the blade.

- $\lambda$  is the tip-speed-ratio [rad], i.e. the ratio between tip speed and wind speed, expressed as:

$$\lambda = \frac{v_{tip}}{U} = \frac{\omega_{wtr} \cdot R}{U}$$

where  $U$  is the wind speed.

- $\theta$  is the blade pitch angle [deg], i.e. the angle between the chord line and the tip speed (see top view in Figure 2)
- $\alpha$  is the angle of attack [deg], i.e. the angle between the chord line and the relative wind  $W$ . Notice that the size of the angle of attack is influenced by the wind speed  $U$ , the turbine rotational speed  $\omega_{wtr}$  and the blade pitch angle  $\theta$ .

As illustrated in Figure 2, in the top view, the two main forces acting on a blade are:



- Lift force  $L$  – which is perpendicular to the direction of motion, i.e. to the relative wind speed  $W$  acting on the blade. This force is desired to be large.
- Drag force  $D$  - which is a force parallel to the direction of motion. This force should be small.

According to [1], these two forces are quadratic functions of the relative wind speed  $W$ . Besides this, their size is directly proportional with the lift and drag coefficient, respectively. These coefficients are given by the manufacturers as functions of the angle of attack  $\alpha$ . This means that, for a given airfoil, the wind speed and the angle of attack dictate the size of the lift and drag forces.

## Power coefficient

The aerodynamic power extracted from the wind by a wind turbine is typically computed based on the power coefficient  $C_p$ , which is non-dimensional and reflects the efficiency of the aerodynamic rotor.

The maximal value of the power coefficient of any wind turbine cannot exceed the theoretical Betz limit of  $C_p^{Betz} = 16/27 = 0.59$ , obtained for stationary operation of wind turbines [1]. This means that a wind turbine can only convert less than 59% of the kinetic energy in the wind into mechanical energy.

For any airfoil profile, the power coefficient is function of the wind speed  $U$ , the rotor speed  $\omega_{wtr}$  and the blade pitch angle  $\theta$ , as follows:

$$C_p = C_p(U, \omega_{wtr}, \theta) \quad (1)$$

Notice that the rotor efficiency of a wind turbine can be controlled by adjusting, if possible, its rotor speed  $\omega_{wtr}$  and pitch angle  $\theta$ .

In practice, it is commonly used to express the power coefficient as function of two variables, i.e. tip-speed-ratio  $\lambda$  and the pitch angle  $\theta$ . The dependency of the wind speed  $U$  and rotor speed  $\omega_{wtr}$  is thus incorporated into the expression of the tip-speed-ratio  $\lambda$ :

$$\lambda = \frac{v_{tip}}{U} = \frac{\omega_{wtr} \cdot R}{U} \quad \Rightarrow \quad C_p = C_p(\lambda, \theta) \quad (2)$$

Figure 3 illustrates a three dimensional graph of the power coefficient, as function of tip-speed-ratio and pitch angle. Notice that the maximum efficiency of the profile  $C_p^{max}$  in extracting power out of the wind can be reached for one certain pitch angle and tip-speed-ratio. This corresponds to an optimal angle of attack, which yields to the highest lift to drag ratio.

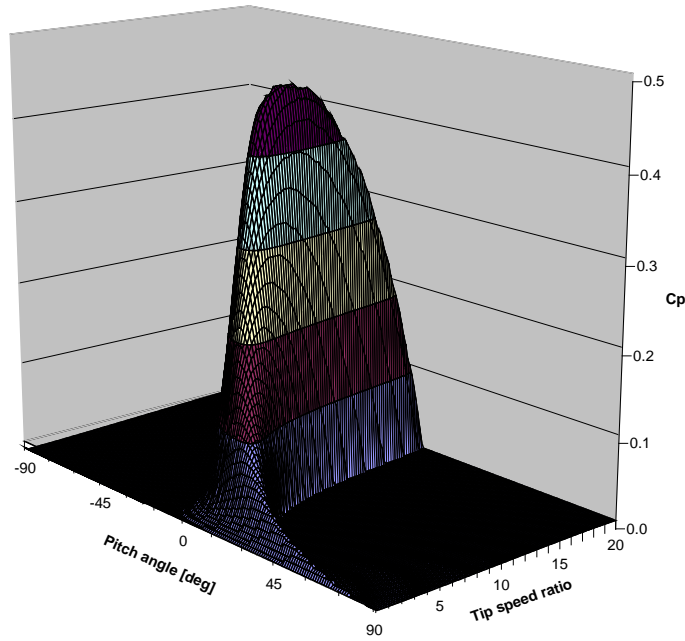


Figure 3: Power coefficient  $C_p$  as function of tip-speed-ratio and pitch angle.

By feathering the three dimensional graph, illustrated in Figure 3, a cross sectional view of the power coefficient for a certain tip-speed-ratio or pitch angle can be obtained, as illustrated in Figure 4. Notice that for each situation an optimal power coefficient  $C_p^{opt}$  can be achieved for a corresponding value  $\lambda_{opt}$  or  $\theta_{opt}$ , respectively.

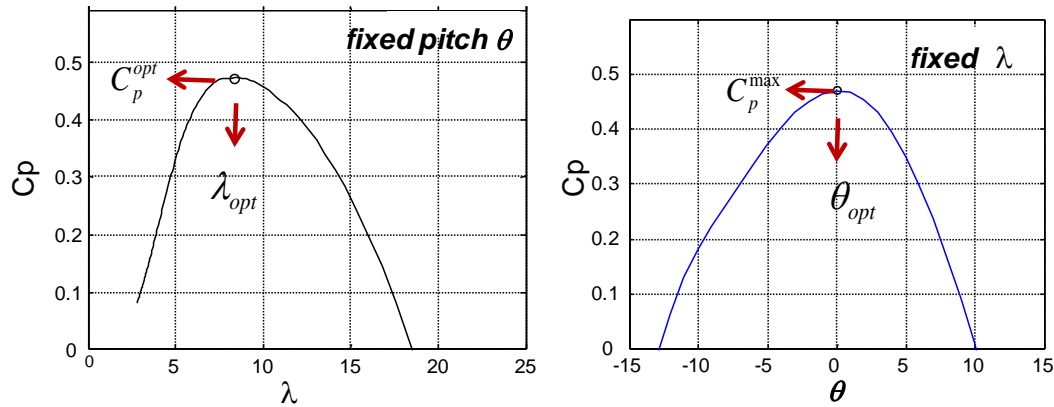


Figure 4: Power coefficient  $C_p$  as function of tip-speed-ratio or pitch angle.

The graph in the left side of Figure 4 corresponds to the situation of wind turbines with fixed speed  $\omega_{wtr}$  and fixed pitch angle  $\theta$ . Notice that such wind turbines can operate with an optimum efficiency  $C_p^{opt}$  at an optimal tip-speed-ratio  $\lambda_{opt}$ , which can be achieved at only one particular wind speed.

The graph in the right side of Figure 4 corresponds to the variable speed wind turbines, where the wind turbines can operate with maximum efficiency  $C_p^{max}$  over a wide range of wind speeds by tracking the optimal

tip-speed-ratio  $\lambda_{opt}$ , i.e. by continuously adapting the rotor speed  $\omega_{wtr}$  of the wind turbine to the wind speed according to:

$$\omega_{wtr} = \frac{\lambda_{opt} \cdot U}{R} \quad (3)$$

These wind turbine concepts will be explained in further detail later in the document.

## *Aerodynamic power*

Once the power coefficient of the aerodynamic rotor is defined, the aerodynamic power of a wind turbine in stationary operational conditions can be expressed as follows:

$$P_{wtr} = \frac{1}{2} \rho A U^3 C_p(\lambda, \theta) \quad (4)$$

Notice that the aerodynamic power is a function of wind speed, air density, rotor size and efficiency. It is proportional to the rotor area  $A$  (i.e. to  $R^2$ ) and to the cube of the wind speed  $U$ .

## *Power curve and operational zones*

Another relevant aspect regarding wind turbine modeling is the power curve. A power curve of a wind turbine is a graph that indicates the relation between the wind speed and the electrical power output of the wind turbine.

Figure 5 illustrates both the aerodynamic power (dashed line) and the power curve (solid line) of a wind turbine. The aerodynamic power curve reflects the power expressed in (4), i.e. how much power would be possible to extract from the wind, if no physical restrictions are imposed. The power curve represents the power, which is actually produced by the wind turbine as result of wind turbine control and limitations. These limitations are imposed at high wind speeds in order to prevent overloading of the wind turbine. The limited power in the power curve at high wind speeds, called rated power, is a turbine design value provided by the manufacturer reflecting the maximum power, which is allowed to be produced by the turbine.

The rated wind speed, also indicated in Figure 5, is the lowest wind speed, at which the produced power of a wind turbine reaches the rated power value. The cut-in and cut-out speeds define the operating range of the wind turbine, where it is assured that the available energy is above a minimum threshold and structural health of the turbine is maintained.

As illustrated in Figure 5, the power curve has two distinct regions:

- Power optimization zone for wind speeds lower than rated wind speed, where the wind turbine is typically designed to produce an optimal power, namely to have maximum efficiency in extracting power out of the wind.

- Power limitation zone for wind speeds higher than rated wind speed, where the wind turbine is designed to limit the power production to rated power value.

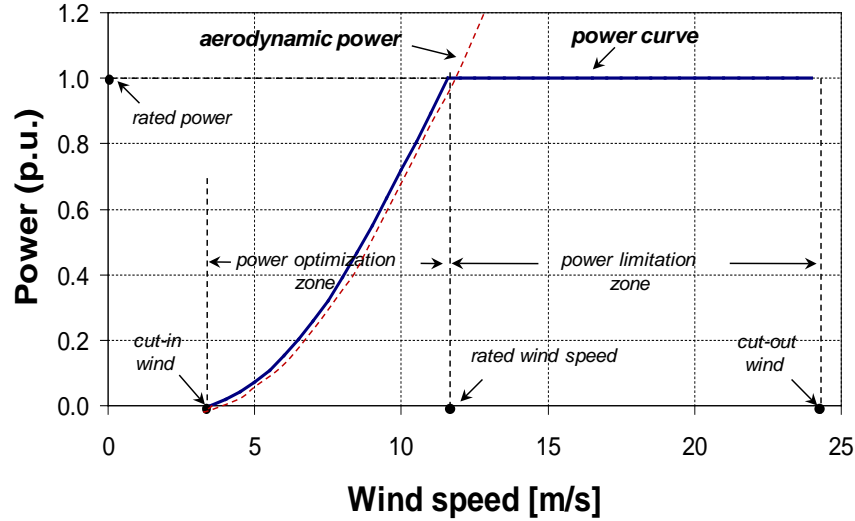


Figure 5: A wind turbine power curve example.

As indicated in Figure 5, it is important to be able to control (i.e. to optimize and limit) the power produced by a wind turbine. This can be done through different control methods, mainly by adjusting the rotor speed  $\omega_{wtr}$  and the blade pitch angle  $\theta$ . These methods will be presented later on in the document.

## 2. Main wind turbine components

The scope of this section is to describe briefly the main components wind turbines, illustrated in Figure 6. As the aerodynamic rotor has already been addressed in the previous section, focus will only be in the following on the generator, the gearbox and the power electronic. It is not the task here to provide a detailed description of the components technology. The purpose is to present basic knowledge on wind turbine modelling issues, which are relevant to be used in power system frequency control studies.

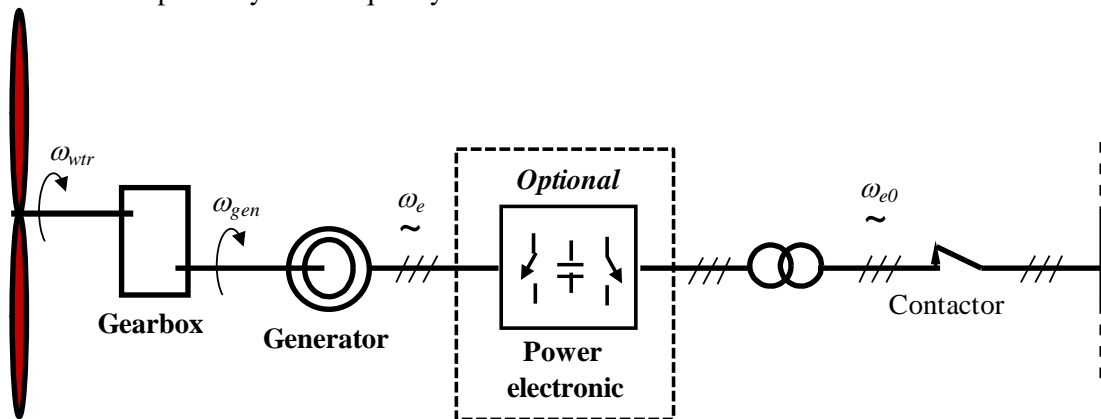


Figure 6: Main components in a wind turbine.

## Generator

As previously mentioned, the generator of a wind turbine converts the mechanical power from the wind into electrical power, which is then fed into the power system. Basically, a wind turbine can be equipped with any type of three-phase generator, i.e. synchronous or asynchronous (induction) generators.

In Figure 6, the electrical angular speed  $\omega_{e0}$  on the grid is expressed as:

$$\omega_{e0} = 2 \cdot \pi \cdot f_0 \quad [rad / s] \quad (5)$$

where  $f_0$  is the grid synchronous frequency, i.e. 50 Hz in Europe and 60 Hz in USA. The synchronous generator speed  $\omega_{gen0}$  is determined jointly by the number of polepairs  $N_{pp}$  of the generator and the electrical angular speed  $\omega_{e0}$  as follows:

$$\omega_{gen0} = \frac{\omega_{e0}}{N_{pp}} = \frac{2 \cdot \pi \cdot f_0}{N_{pp}} \quad [rad / s] \quad (6)$$

Its expression in rotations per minute [rpm] is:

$$n_{gen0} = 60 \cdot \frac{f_0}{N_{pp}} \quad [rpm] \quad (7)$$

The rotor of an asynchronous generator rotates at a slight different speed than the synchronous generator speed  $\omega_{gen} \neq \omega_{gen0}$ . This leads to the fact that an electric field is induced between the rotor and the rotating stator field by a relative motion, known as slip, which causes a current in the rotor windings [2],[3]. The slip is defined as:

$$s = \frac{\omega_{gen0} - \omega_{gen}}{\omega_{gen0}} \cdot 100 \quad [\%] \quad (8)$$

The rotor speed of an asynchronous generator may vary according to the slip variations, which for a fixed speed wind turbine are typically in range of 1-2%. The design of the generator speed is in general thus dependent on the electrical angular speed  $\omega_{e0}$ , the generator slip  $s$  and the generator polepairs  $N_{pp}$  according to:

$$\omega_{gen} = \frac{\omega_{e0}}{N_{pp}} (1 - s) \quad (9)$$

In **frequency control studies**, which are the scope for the wind turbine modeling presented in this document, the effect of the slip (for fixed speed wind turbines) is not significant (i.e.  $s \approx 0$ ) and it can be therefore assumed that in such studies the generator speed equals the synchronous generator speed, namely:

$$\omega_{gen} = \omega_{gen0} = \frac{\omega_{e0}}{N_{pp}} \quad (10)$$

This expression shows also that by increasing the number of polepairs  $N_{pp}$ , the generator speed is decreasing.

## Gearbox

The gearbox of a wind turbine converts the slow high torque rotation of the turbine rotor into the much faster rotation of the generator. The gearbox ratio of a gearbox is the relationship between the high speed of the generator  $\omega_{gen}$  and the low speed of the turbine rotor  $\omega_{wtr}$ , expressed as follows:

$$N_{gear} = \frac{\omega_{gen}}{\omega_{wtr}} \quad (11)$$

As the generator speed decreases by increasing number of polepairs, the gearbox may not be necessary for multipole wind turbine generator systems, i.e. where number of polepairs may be higher than 100.

## Power electronic

As indicated in Figure 6, the presence of power electronics in the wind turbines may be optional depending on the type of wind turbine configuration, namely if it is with or without speed control (fixed or variable speed wind turbines).

In the fixed speed wind turbines, which are directly connected to the grid, simple and cheap electric components are typically used in order to limit the disturbances, the connection of the turbine to the grid inserts into the grid. Meanwhile, more advanced power electronics are used to connect variable speed wind turbines. Since such wind turbines operate at variable rotational speed, the electric frequency of their generator varies and must therefore be decoupled from the frequency of the grid. This can be achieved by using a power electronic converter system, which is a device able to interconnect two electrical systems with different and independent frequencies.

The presence of power electronics inside wind turbines offers enlarged grid friendly control capabilities and therefore it plays a vital role in the integration of large future wind farms.

## 3. Wind turbine concepts

The scope of this section is to present the most commonly applied wind turbine designs in the industry today.

Wind turbine technology has matured during the years with a rapid development of the different wind turbines concepts [4]. These concepts are characterized by different electrical designs and controls, and they can be classified according to their **speed control** and **pitch control** ability, as follows:

- Speed control ability criterion divides the wind turbines into:
  - Fixed speed wind turbines
  - Variable speed wind turbines
- Pitch control ability criterion classifies the wind turbines into:
  - Fixed pitch wind turbine concepts
    - Passive stall control wind turbines
  - Variable pitch wind turbine concepts
    - Pitch control wind turbines
    - Active stall control wind turbines

The main differences between all these wind turbine concepts concern the generating system and the way in which the efficiency of the aerodynamic rotor is controlled, i.e. optimised at low wind speeds and limited at high wind speeds in order to prevent overloading of the wind turbine.

Table 2 summarises the control characteristics of these wind turbine concepts, which will be explained in more details in the next sections. Notice that fixed speed wind turbines (with fixed  $\omega_{wtr}$ ) are generally either

passive or active stall controlled wind turbines. Pitch control applied on fixed speed wind turbines is not an attractive solution, because the pitch mechanism is not fast enough to avoid large power fluctuations in case of gusts at high wind speeds. The fixed speed wind turbine with passive stall control is considered in the following.

Table 2: Overview on the most common wind turbine concepts on the market today.

Wind turbine concept	Rotational speed $\omega_{wtr}$	Pitch angle $\theta$
<i>Passive stall control wind turbine</i>	fixed	fixed
<i>Active stall control wind turbine</i>	fixed	variable
<i>Pitch control wind turbine</i>	variable	variable

Notice also that in contrast to fixed speed wind turbines, variable speed wind turbines are today almost exclusively used in combination with variable pitch [5]. The variable speed operation assures, that sudden energy surplus of wind gusts is temporarily buffered as rotational energy in the wind turbine rotor until the pitch control limits the excess power.

Figure 7 illustrates by comparison the power curves of passive stall wind turbine, active stall wind turbine and pitch controlled wind turbines. It can be seen that the power may be smoothly limited by rotating the blades either by pitch or by active stall control, while the power limitation of the stall control presents an overshoot. Figure 7 reflects thus how the rotor efficiency of a wind turbine can be controlled by adjusting, if possible, the pitch angle and the rotor speed.

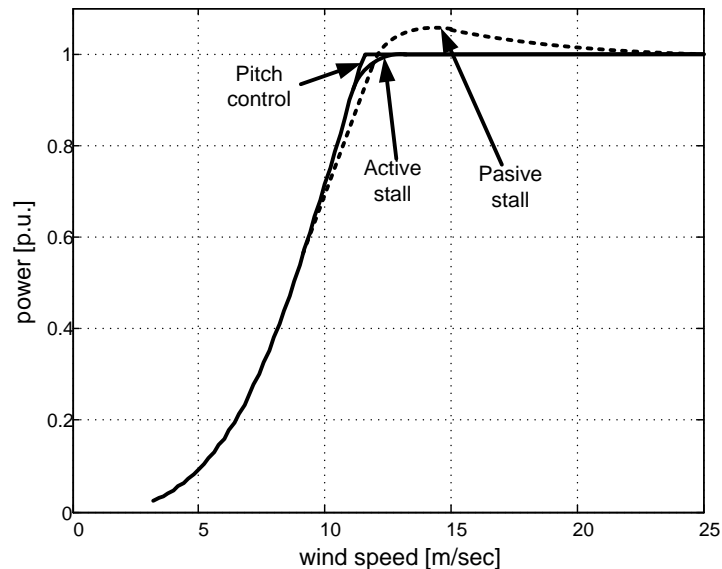


Figure 7: Power curves for the three wind turbine concepts: passive stall control, active stall control and pitch control wind turbines.

In the following sections, the modelling and control issues of fixed and variable speed wind turbines will be presented in further details especially with focus on the wind turbine concepts underlined in Table 2.

### 3.1 Fixed speed wind turbines (FSWTs)

Fixed speed wind turbines (FSWTs) were the most installed wind turbines in the early 1990s. They were dominating the market until the mid 1990s, when the size of wind turbines started to be in MW range. Characteristic for fixed speed wind turbines is that they are equipped with an asynchronous (induction) generator connected directly to the grid, as illustrated in Figure 8. This means, that regardless of the wind speed, the wind turbine rotor speed is almost fixed. It is fixed to the grid frequency and cannot be changed. In this respect, the grid behaves like a large flywheel holding the rotor speed of the turbine nearly constant irrespective of changes in wind speed.

FSWTs have the advantages of being simple, robust, reliable and well proven and using low-cost electrical parts. Their direct drawbacks are high mechanical stress and limited power quality control [6]. It is worth noticing that for such wind turbine, due to its fixed speed operation practically all fluctuations in the wind speed are directly transmitted as fluctuations in the electrical power to the grid.

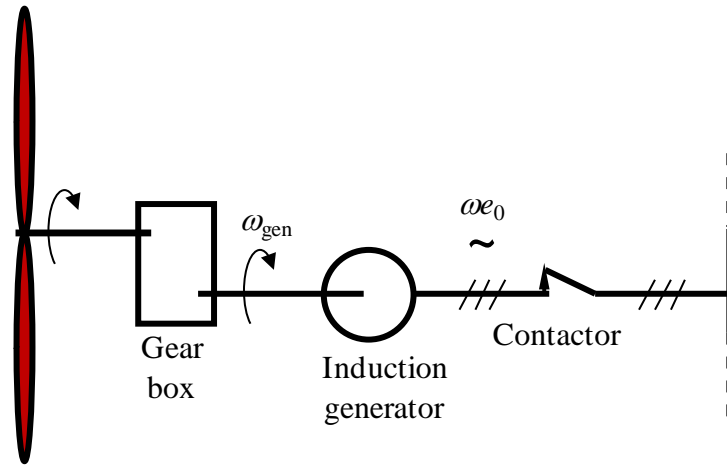


Figure 8: Fixed speed wind turbine – directly connected to the grid.

FSWTs are able to operate with maximum aerodynamic efficiency at only one particular wind speed. They are therefore designed to operate with maximum aerodynamic efficiency at the most likely wind speed in the area, the wind turbine is placed. This means that in the optimization design algorithm, the power efficiency must reach a maximum at the most likely wind speed  $U_{opt}$  in the installation area. The optimised power can be then expressed as:

$$P_{opt} = \frac{1}{2} \rho \pi R^2 U_{opt}^3 C_p^{opt} \quad (12)$$

Once knowing the most likely wind speed  $U_{opt}$  in the installation area and the fixed pitch angle at which the wind turbine blades are mounted on the hub, the optimal tip-speed-ratio can be determined from the power



coefficient curve, as illustrated in Figure 4. Based on  $\lambda_{opt}$ , a wind turbine designer can then compute the wind turbine rotational speed, as follows:

$$\omega_{wtr} = \frac{\lambda_{opt} \cdot U_{opt}}{R} \quad (13)$$

Notice that, by keeping unchanged the aerodynamic characteristics, one way to increase the power of a wind turbine installed in a certain area is to increase the rotor area, i.e. rotor radius  $R$ . This means that, in order to keep the tip-speed-ratio fixed to its optimal value  $\lambda_{opt}$ , the rotational speed of the turbine  $\omega_{wtr}$  has to be reduced.

Another way to increase the absorbed energy from the wind by a fixed speed wind turbine is to make it to operate at two different fixed rotational speeds. This may be realised by equipping the wind turbine with an asynchronous generator with changeable number of polepairs, i.e. with two winding sets: one used at low wind speeds and the other at medium and high wind speeds. The idea is that running the turbine slower by using a higher number of polepairs makes the turbine to be more effective at low wind speeds.

Figure 9 illustrates an example of how a generator can be geared by changing the number of polepairs, i.e. by increasing number of polepairs  $N_{pp}$ , the generator speed is decreased, as expressed in (10).

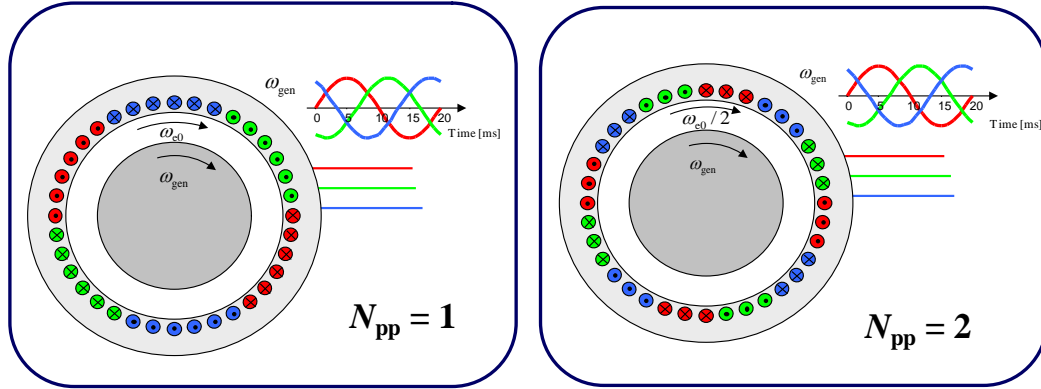


Figure 9: Changeable pole pairs.

For example, at low wind speeds, where the turbine has a better energy capture performance for smaller rotational speeds, the winding set with a number of polepairs of  $N_{pp}=3$  is used. On the other hand, at high wind speeds, where the turbine has a better energy capture performance for higher rotational speeds, the winding set with a number of polepairs of  $N_{pp}=2$  is used.

Figure 10 sketches the algorithm to compute the gear ratio of a FSWT. Notice that the decision for the size of the gear ratio is based on the trade-off between the optimization design of the rotor speed and the design of generator speed. On one side of the gearbox, the rotor speed is designed based on the optimization algorithm of the rotor efficiency, while on the other side the generator speed is defined based on the generator characteristics.

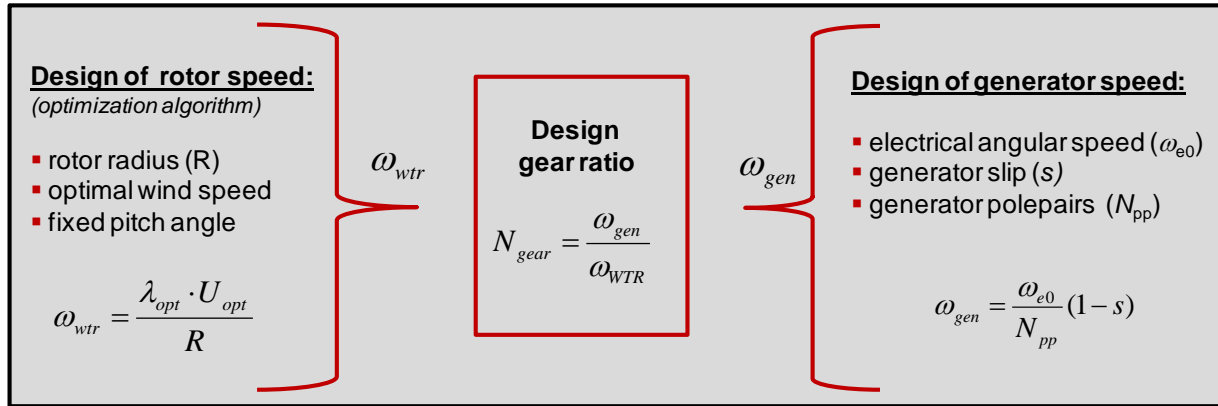


Figure 10: Design of gear ratio for a fixed speed wind turbine.

Once the gear ratio is defined, the rotor speed can be rewritten as follows:

$$\omega_{wtr} = \frac{\omega_{gen}}{N_{gear}} = \frac{2 \cdot \pi \cdot f}{N_{pp} \cdot N_{gear}} = \frac{2 \cdot \pi \cdot f_0}{N_{pp} \cdot N_{gear}} \cdot (1 - s) \xrightarrow{\text{and if assuming } s \approx 0} \omega_{wtr} \approx \frac{2 \cdot \pi \cdot f_0}{N_{pp} \cdot N_{gear}} \quad (14)$$

It is thus worth noticing that the rotor speed of a fixed speed wind turbine depends on the frequency of the connected grid, generator design (slip and pole pairs) and gear ratio. This aspect is very interesting and important to keep in mind especially when wind turbines are planned to be moved from one grid to another grid with a different synchronous frequency, i.e. from 50Hz to 60Hz (from Europe to USA). In these situations, the designer should keep in mind to adjust adequately the gear ratio of the turbine too.

The earliest and most simple method of limiting the power output at high wind speeds is the so-called passive stall control. Passive stall control wind turbines are fixed speed wind turbines with rotor blades firmly attached to the hub at a fixed pitch angle. As the pitch angle and rotor speed are fixed, the power and power coefficient of such wind turbines can be expressed only as function of wind speed  $U$ , as follows:

$$P_{wtr} = \frac{1}{2} \rho A U^3 C_p(U) \quad (15)$$

Passive stall controlled wind turbines have normally a fixed speed rotor or a two speeds rotor, the last being achieved, as mentioned before, by using a generator with two different numbers of polepairs.

Figure 11a illustrates the lift coefficient versus the angle of attack for a passive stall wind turbine. It is worth noticing that for such wind turbines, where the pitch angle and the rotation speed are fixed, the angle of attack of the blades, defined in Figure 2, depends only on the wind speed. This means that, an increase in the wind speed leads to an increase of the angle of attack. Notice in Figure 11a how the rotor ‘automatically’ loses the aerodynamic efficiency (known as stall phenomena), when the wind speed and thus the angle of attack exceeds a certain critical level. The dotted line in Figure 11a indicates the operational regime of the passive stall wind turbine, i.e. at low angles of attack (corresponding to low wind speeds) the lift coefficient increases proportionally until the critical angle of attack (about 15 deg) is reached, while at high wind speeds (angles of attack higher than 15deg) the lift coefficient starts to decrease implying the blade enters the stall.

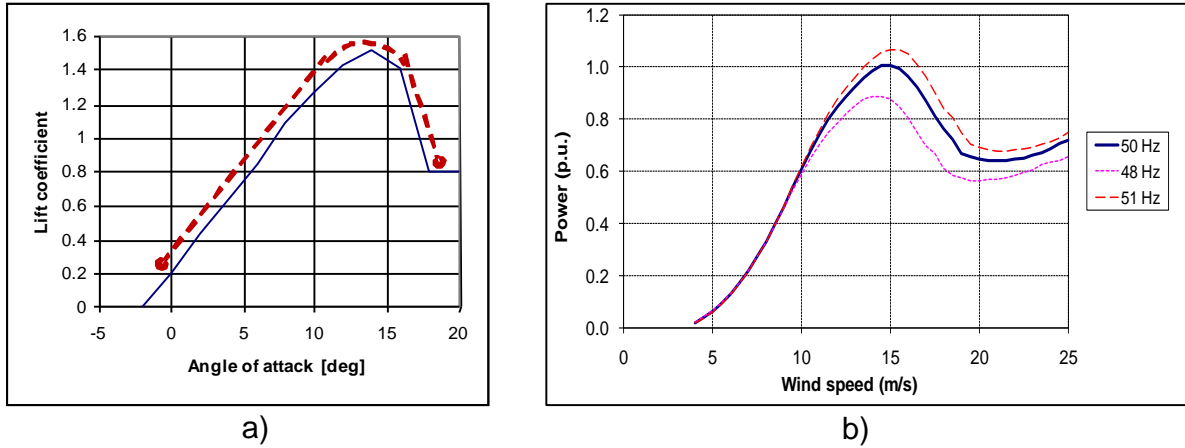


Figure 11: a) The lift coefficient versus angle of attack. b) The power curve for a passive stall wind turbine for three given grid frequencies.

The influence of the wind speed and indirectly of the angle of attack on the aerodynamic efficiency of the rotor is also reflected in the power curve of the passive wind turbine, as illustrated in Figure 11b. Moreover, Figure 11b shows also the power curves of such wind turbines for different grid frequencies. Notice how the grid frequency influences the power curve of a passive stall wind turbine, i.e. a higher grid frequency implies a large wind turbine power production and vice-versa.

Some drawbacks of the passive stall control are high mechanical stress caused by wind gusts and variations in the maximum steady state power due to variation in the air density, dirt on blades and grid frequencies. The advantage of passive stall wind turbines is however that their design is very simple and cheap, since no bearings or pitch mechanism is needed for the blades.

Figure 12 shows the model block diagram of a passive stall wind turbine. Once the fixed pitch angle and the generator speed are decided, the power produced by the wind turbine at different wind speeds can be computed using (15).

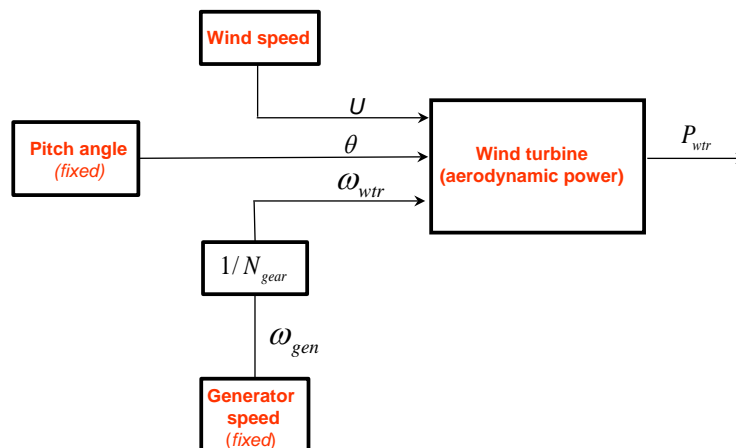


Figure 12: Model block diagram of a FSWT - passive stall wind turbine concept (in physical unit).

Figure 13 provides an overview of the passive stall wind turbine concept. It illustrates the characteristic curve shapes of the passive stall wind turbine concerning power, rotor speed, pitch angle and aerodynamic efficiency versus wind speed.

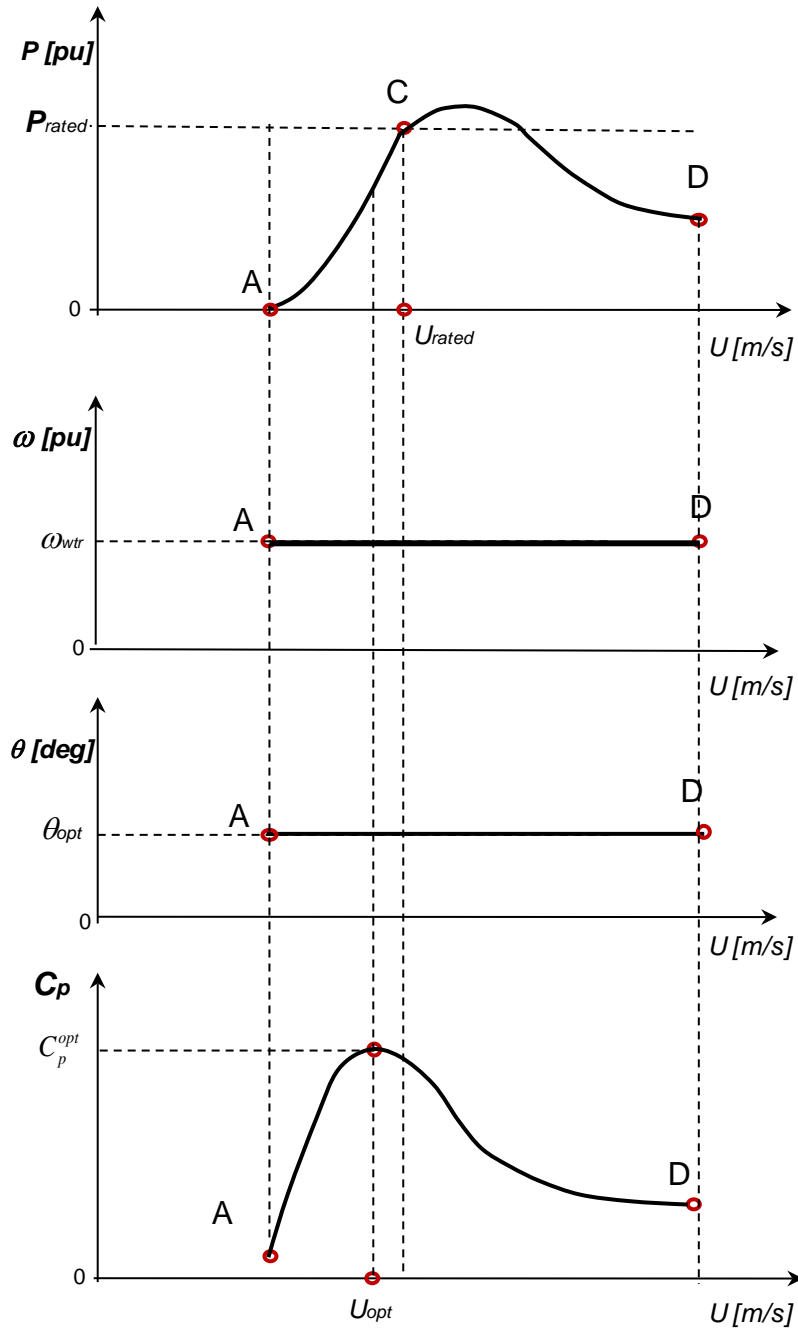


Figure 13: FSWT - Passive stall control wind turbine concept.

Notice that the limitation of the power is not very accurate. The turbine enters automatically the stall regime at high wind speeds. Moreover it presents a big overshoot in power compared to the rated power. Figure 13 shows also the rotor speed and the pitch angle, which are kept constant independent of the wind speed. The wind

turbine only operates at maximum efficiency at one wind speed in the low-speed region. Figure 13 reveals thus the poor power regulation of such type of wind turbine, as a result of its constrained operation conditions, i.e. no possibility to change/control neither the pitch angle nor the rotor speed.

### 3.2 Variable speed wind turbines (VSWTs)

Variable speed wind turbines (VSWTs) have become the most dominant type among the installed wind turbines during the last 10 years [5]. Characteristic for variable speed wind turbines is that they are designed to operate with variable rotational speed. This means that the electric frequency of their generator varies too and must therefore be decoupled from the frequency of the grid. This is typically achieved by connecting the turbine to the grid through a power electronic interface (i.e. power converter), as it is illustrated in Figure 14.

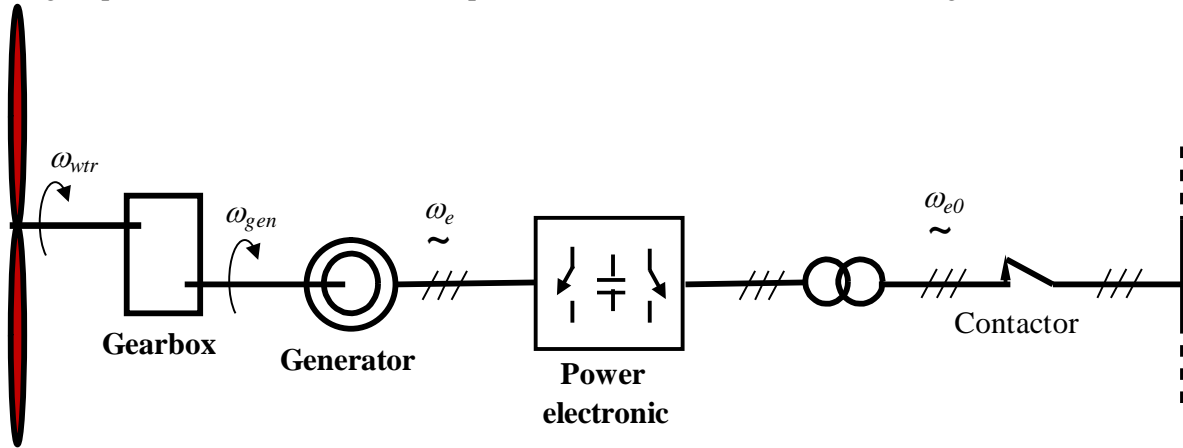


Figure 14: Variable speed wind turbine connection to the grid through a power electronic interface.

The presence of power electronics makes the variable speed operation itself possible. The primary task of a power converter is to control the speed of the generator. Besides this, the power converter provides also several control capabilities for variable speed wind turbines, like control of power, frequency and voltage on the grid, enabling them to behave as active components on the grid similar to the traditional power plants. With regards to control performance variable speed wind turbines are faster than the power plants, but of course their power production still depends on the available wind resource.

It is not the task here to explain the many different power converter technologies existing today in different variable speed wind turbines concepts. The task is to present basic issues related to the modelling and control strategy of variable speed wind turbines. It can however be shortly mentioned that there are two groups of variable speed wind turbine concepts, which are dominating the market today. The first concept is connected to the grid through a partial scale frequency converter (known as doubly-fed induction generator DFIG wind turbine concept), while the second concept is connected to the grid through a full scale frequency converter. Plenty of detailed publications about this subject can be found in the literature.

By introducing the variable speed operation, it is possible to continuously adapt (accelerate or decelerate) the rotational speed of the wind turbine to the wind speed, in such a way that the turbine operates continuously at its highest level of aerodynamic efficiency. Contrary to FSWTs, which are designed to obtain maximum efficiency at one wind speed only, the VSWTs are thus designed to achieve maximum aerodynamic efficiency over a wide range of wind speeds.

The prime target of variable speed wind turbines at lower wind speeds is to adjust the rotor speed at changing wind speeds so that the power efficiency  $C_p(\lambda, \theta)$  always is maintained at its maximum value. The power coefficient  $C_p(\lambda, \theta)$  has a maximum for a particular tip-speed-ratio  $\lambda_{opt}$  and pitch angle  $\theta_{opt}$ . This means that for extracting maximum power from a particular wind speed, the control strategy has to change the turbine

rotor speed in such a way that the optimum tip-speed-ratio  $\lambda_{opt}$  always is obtained. The maximum power a particular wind turbine can extract from the wind is a cubic function of the turbine optimum speed, as follows:

$$P_{\max} = K_{opt} [\omega_{wtr}^{opt}]^3 \quad (16)$$

where:

$$K_{opt} = \frac{1}{2} \rho \pi R^5 \frac{C_p^{\max}}{\lambda_{opt}^3} \quad (17)$$

where  $K_{opt}$  depends on the rotor characteristics and the air density. Tracking the maximum power is the goal as long as the generated power is less than the rated power. At wind speeds higher than rated wind speed, the control strategy is changed so that the wind turbine no longer produces maximum power but only rated power. The blades have thus to be pitched to reduce the power coefficient and thereby to maintain the power at its rated value. The adjustment of the pitch angle keeps the rotor speed constant, preventing over loading of the turbine at high wind speeds. In both operational modes (i.e. power optimization and limitation) variable speed wind turbines achieve a better energy capture compared to fixed speed wind turbines [2].

Notice however that an increase of the turbine rotational speed at a certain wind speed implies automatically an increase of the tip-speed-ratio and thus also of the aerodynamic noise generated by the rotor. In order to avoid this, the tip-speed-ratio  $\lambda$  of modern wind turbines is typically limited implying a maximum tip-speed-ratio of approximately 70m/s.

Besides an improved power production, VSWTs also have an improved power quality, a reduced mechanical stress (as wind speed variations are absorbed as variations in the rotor speed), a reduced acoustical noise and, last but not least, a high control capability. This is of high relevance for the integration of large wind farms in the power system. Owing to these advantages, variable speed operation is the predominant choice for MW-scale turbines today. Its direct drawbacks are additional losses and increased capital cost due to the power electronics. However, it can be expected, that costs on power electronics will further decrease in the future due to improved developments.

The main trend of future wind turbine design is using variable speed operation, due to its very attractive features. The presence of power electronics makes it possible for wind turbine to handle the grid requirements imposed by system operator. It is therefore obvious that power electronics will continue to play a vital role in the development of wind turbines and in the integration of future large wind power within the electrical supply system.

As already sketched in Table 2, due to power limitation considerations, VSWTs are exclusively used in combination with variable pitch control. The pitch control concept, as it will be explained in the following section, is more attractive than the active stall control concept. For example, the pitch control is strongly preferred particularly for the large machines (in MW range), as there is some concern about stall induced vibrations (namely vibrations which occur as the blade enters stall).

Today's most widespread power limitation control method is pitch control, where the blades can be quickly turned out or into the wind, as the power output becomes too high or too low, respectively. Pitch angle adjustment is the most effective way to limit the output power of a wind turbine, due to its fast effect on the aerodynamic forces on the blade at high wind speeds.

Pitch control wind turbines are today, as already mentioned, used in combination with the variable speed operation. They have the ability to adjust their rotor speed and blade angle in a controlled manner. Nowadays pitch control with fixed speed operation is not used anymore, primarily due to large power fluctuation at high wind speeds.

In Figure 15a, it is shown in which direction the pitch angle of a pitch controlled wind turbine is adjusted. The idea is to turn the blades out of the wind at high wind speeds. For the sake of a quick understanding of the pitch control graph in Figure 15a, it is easy to assume that the rotor speed of the turbine is constant. In this condition, notice that for a certain wind speed  $U$  and an assumed fixed rotor speed (i.e. constant tip speed  $v_{tip}$ ), the vector of the relative wind speed  $W$  (wind speed seen by the airfoil) keeps its amplitude and direction constant. By increasing the pitch angle in the direction indicated in Figure 15a, i.e. by moving the airfoil nose against the incoming wind  $U$ , the angle of attack  $\alpha$  is decreasing, while the chord line is coming closer to the relative wind speed. This means that according to the lift characteristic of the airfoil, the lift coefficient is also decreased, as illustrated in Figure 15b. This phenomenon explains the reduction of the aerodynamic power when the pitch angle is increased. The dotted line in Figure 15b shows the action of the pitch control. The idea of pitch control is thus to move the pitch angle in such a way which maintains a fairly laminar flow around the blades for the whole wind speed operational range, while keeping the angle of attack under its critical value.

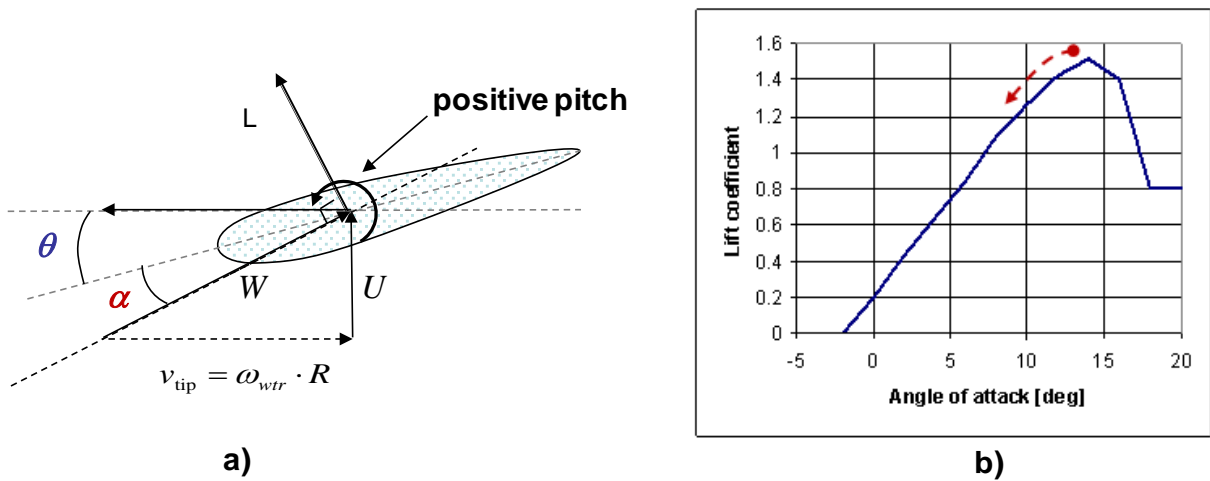


Figure 15: a) Positive pitch direction b) Lift coefficient for a pitch control wind turbine.

Figure 16 illustrates an example of how a pitch control wind turbine is operating. Figure 16 contains several aerodynamic power curves corresponding to different fixed positive pitch angles.

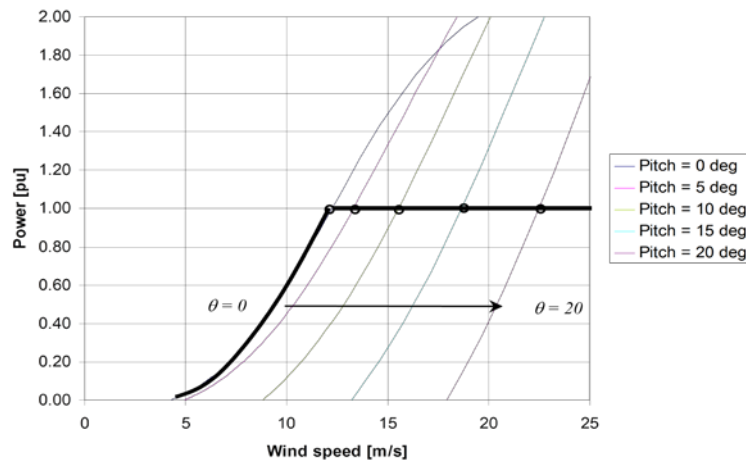


Figure 16: Pitch control wind turbine power curve at different pitch angles.

The bolded curve in Figure 16 corresponds to the actual controlled power curve, which is the result of changing adequately the pitch angle of the wind turbine for given wind speeds in order to limit the power production at high wind speeds. Notice that the pitch angle can vary roughly between 0-35 deg, depending on the airfoil.

The main advantages of pitch control wind turbines are better exploitation of the wind during high wind speed, good power control performance, assisted start-up and emergency-stop power reduction. Some disadvantages are the need for a pitching mechanism and large power fluctuations at high wind speeds in case of fixed speed operation.

Figure 17 shows the modelling block diagram of a variable speed pitch control wind turbine. Notice that comparing to both passive and active stall wind turbine models (Figure 12 and **Error! Reference source not found.**, respectively) the modelling of this wind turbine concept contains more modelling blocks.

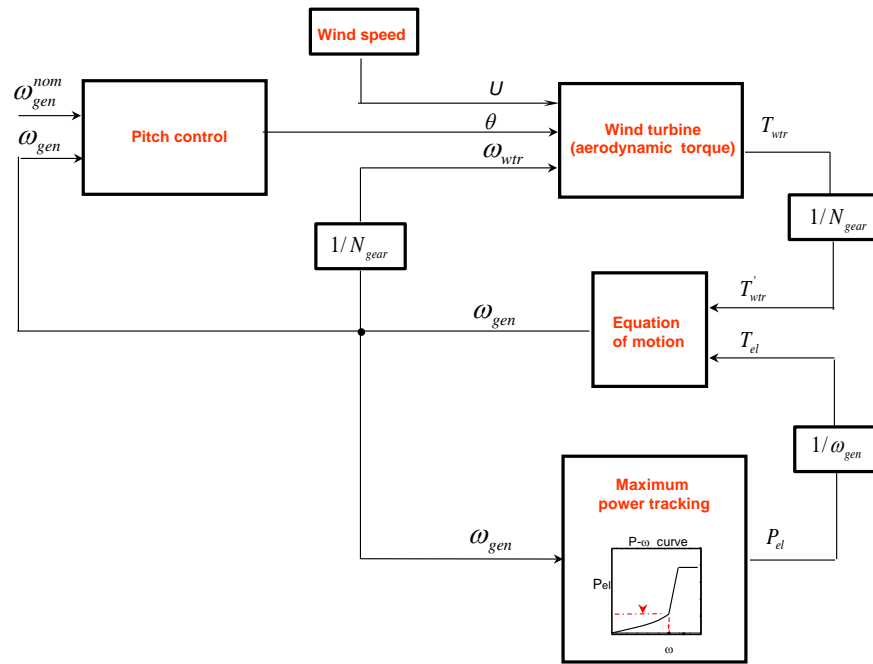


Figure 17: Model block diagram of a VSWT concept (all blocks in physical unit).

The aerodynamic wind turbine model in Figure 17 is based on the aerodynamic power of the turbine, expressed in (4). The output of this block is the aerodynamic torque, calculated as follows:

$$T_{wtr} = \frac{P_{wtr}}{\omega_{wtr}} = \frac{P_{wtr}}{\omega_{gen}} N_{gear} \quad (18)$$

where  $N_{gear}$  is the gear ratio.

The equation of motion can be expressed as follows:

$$T'_{wtr} - T_{el} = I_{echiv} \frac{d\omega_{gen}}{dt} \quad (19)$$



where  $T'_{wtr}$  and  $I'_{echiv}$  are the aerodynamic torque and equivalent inertia on the right side of the gear ratio (i.e. generator side), expressed as follows:

$$T'_{wtr} = \frac{T_{wtr}}{N_{gear}} \quad (20)$$

$$I'_{echiv} = I_{gen} + I'_{wtr}$$

$$I'_{wtr} = \frac{I_{rot}}{N_{gear}^2}$$

where  $I_{gen}$  is the generator inertia and  $I_{rot}$  is the rotor inertia.

The maximum power tracking block inside Figure 17, contains a predefined characteristic (look-up table), i.e. electrical power versus generator speed. Figure 18 illustrates an example of how the maximum power tracking characteristic of a variable speed wind turbine can be designed. Such characteristic is designed based on aerodynamic data of the turbine rotor and its points correspond to the maximum aerodynamic efficiency, limited at high wind speeds. For example the zone *AB* in Figure 18, corresponds to the maximum power presented in the expression (16). As depicted in Figure 17, the maximum power tracking block provides at a given generator speed the electrical power reference for the equation of motion and, in turn, the power that will actually be produced by the wind turbine, neglecting any kind of loss.

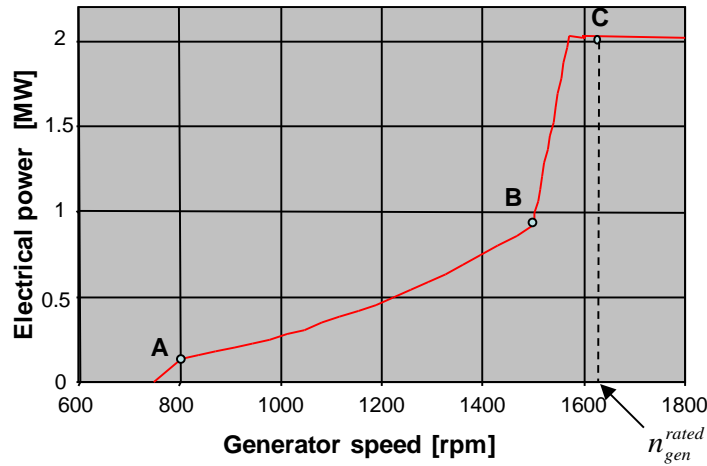


Figure 18: Example of maximum power tracking design for a VSWT.

The pitch control, sketched in Figure 19, has as task to limit the generator speed to its rated value at high wind speeds, i.e. to control the pitch angle in order to prevent the generator speed becoming too large by spilling some of the available aerodynamic power.. At low wind speeds, the pitch angle is kept constant to its optimal value, which for most aerodynamic profiles it is typically close to a pitch angle of zero degrees. For example, the optimal pitch values are typically slightly negative. Although a detailed optimization can be made for wind speeds lower than the rated value, in many situations it is plausible that the factual operational point deviates a little, due to dust and dirt on the blades, oscillations and torsion of the blades. Therefore it can be assumed in terms of generic representation that the optimised pitch value can be replaced by only one value (i.e. 0deg) for the wind speeds below the rated value [2].

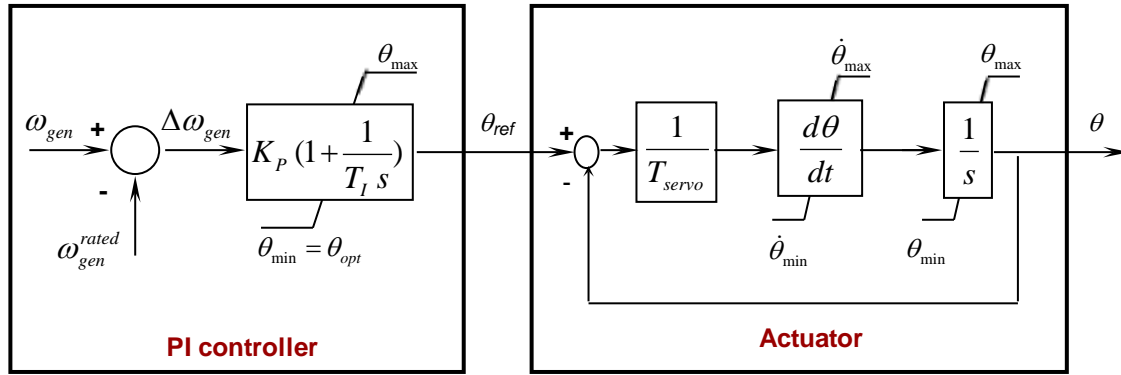


Figure 19: Pitch angle control block from Figure 17.

Notice that the pitch angle control is realised using a PI controller and an actuator with limitation of both the pitch angle and its rate of change. The input of the PI controller is the error signal between the generator speed and its reference value. The PI controller produces the reference value of the pitch angle  $\theta_{ref}$ , which is sent further to the actuator. The actuator compares the measured pitch angle  $\theta$  to the reference pitch angle  $\theta_{ref}$  from the PI controller and corrects the error signal  $\Delta\theta = \theta - \theta_{ref}$ .

At low wind speeds the pitch controller keeps the pitch angle constant to its optimal value, while the tip-speed-ratio is driven to its optimal value by controlling the rotational speed. The difference between the generator speed and its reference value is negative and therefore the controller's output  $\theta_{ref}$  decreases until the controller reaches its lower limit (optimal pitch). At high wind speeds, the pitch control keeps the generator speed limited to its rated value by the control of the pitch angle. The difference between the generator speed and its rated value is now positive and therefore the pitch angle is driven to positive values until the rated generator speed is reached.

Figure 20 provides an overview of the VSWT concept. It illustrates the characteristic curve shapes of the VSWTs concerning power, rotor speed, pitch angle and aerodynamic efficiency versus wind speed.

The VSWTs achieve a better performance compared to fixed speed wind turbines, i.e. an increase in the annual energy production by approximately 5% [2]. Notice how at low wind speeds (i.e. in power optimization) the rotor speed is adjusted continuously in order to achieve and maintain the maximum efficiency  $C_p^{\max}$ . In power optimization the pitch angle is commonly used constant to its optimal value. In power limitation the generator speed is kept constant to its rated value by the pitch controller, which in order to do that, has to increase the pitch angle. Notice also that the increase in the pitch angle implies immediately a decrease in the power efficiency.

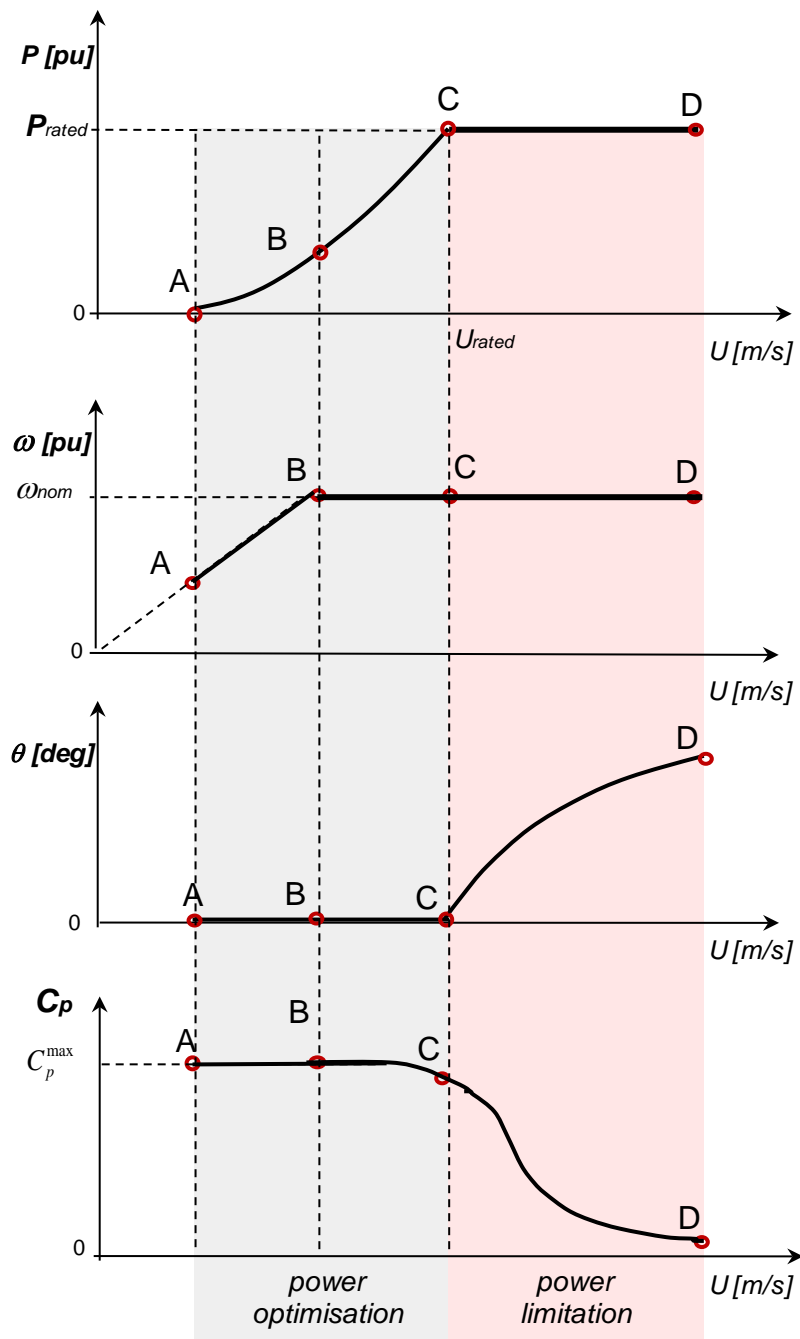


Figure 20: Variable speed wind turbine concept.

### 3.2.1 Introduction on frequency controller of VSWT

Unlike fixed speed wind turbines (FSWTs), variable speed wind turbines (VSWTs) are decoupled from the power system through power electronics. As result of their decoupling from the grid, VSWTs do not inherently contribute to system inertia as FSWTs do. They are therefore not able to sense any frequency deviation in the power system unless they are equipped with a frequency controller.

VSWTs are today preferred to FSWTs as they have higher efficiency and better control options. However if the share of VSWTs in the power system increases, the inertia of the system decreases and this means that the system frequency may drop very rapidly if a power imbalance occurs in the power system. Roughly speaking, a FSWT is able to *feel* a frequency change in the system, while a VSWT is not, since it is completely decoupled from the power system. This means that VSWTs do not release or absorb energy when the system frequency varies.

As illustrated in Figure 21, unlike the FSWT model, that is directly connected to the grid and has therefore as input  $\Delta\omega_{pu}$ , the VSWT needs to get information about the behaviour of the frequency in the system through an additional device. This device is the so called frequency controller, which measures the frequency of the power system and helps thus the VSWT to sense and to react to a frequency deviation.

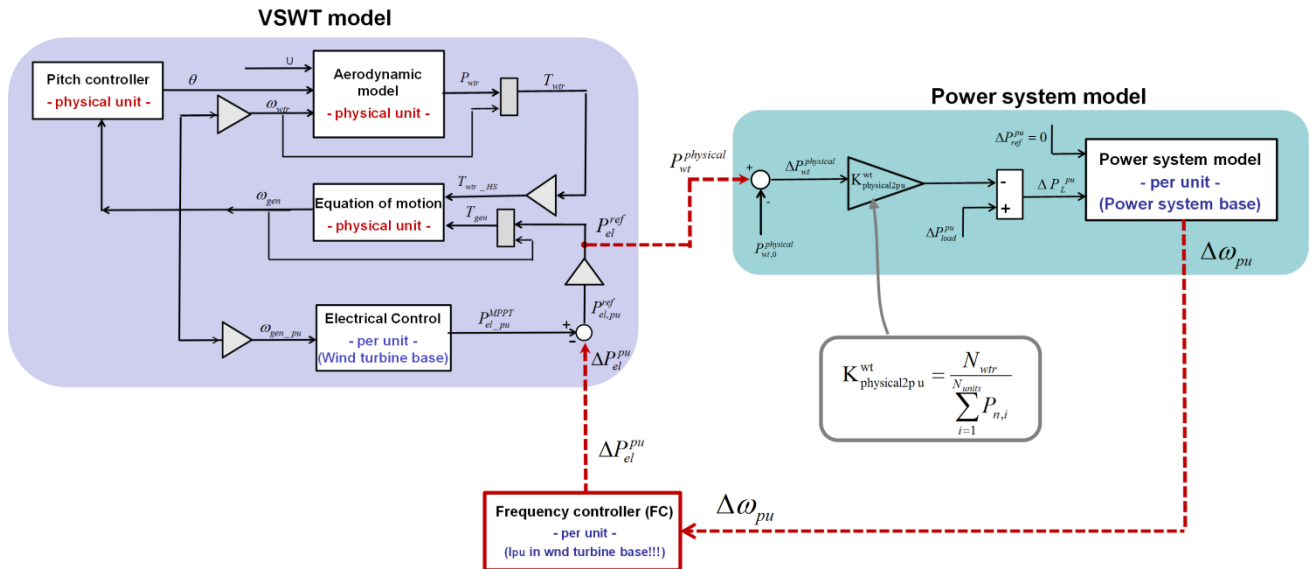


Figure 21: Overview of VSWT's connection to the power system through frequency controller.

Notice that the output of the frequency controller  $\Delta P_{el}^{pu}$  is added to the power output of the Maximum Power point Tracking (MPPT) table. The power reference of the wind turbine is thus corrected according to the frequency deviations in the power system.

The frequency controller can be implemented as a so called **synthetic inertia controller**. Such controller, shown in Figure 22, measures the frequency and emulates a synthetic inertia for the VSWT. The idea of the synthetic inertia controller is to reflect how the kinetic energy stored in the rotating mass of the turbine can be released in case of sudden frequency changes in the system. This means that during a frequency drop, the stored kinetic energy can be exploited by adjusting the power reference of VSWT with  $\Delta P_{el}^{pu}$  according to the frequency changes in the system.

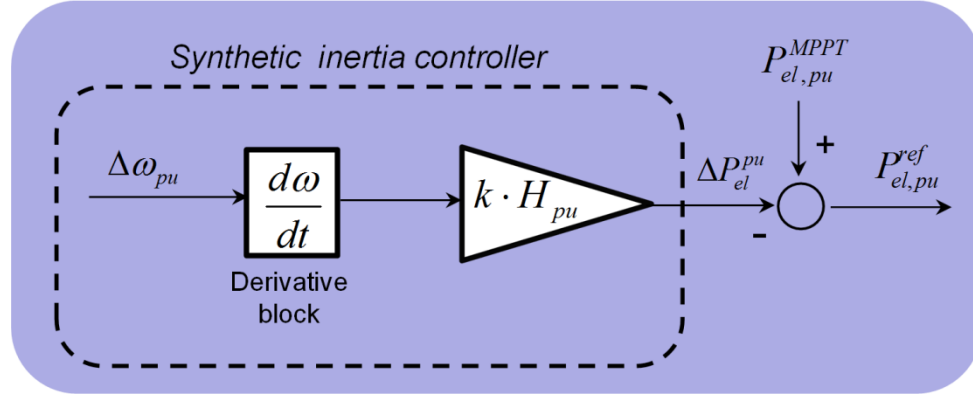


Figure 22: Frequency controller (synthetic inertia controller).

The wind turbine power reference is calculated as following:

$$P_{el,pu}^{ref} = P_{el,pu}^{MPPT} - \Delta P_{el}^{pu}$$

Notice that the power reference for the wind turbine is increased during a frequency drop.

As indicated in Figure 22, the additional power term  $\Delta P_{el}^{pu}$ , i.e. the output of the synthetic inertia controller, is calculated based on information on the rate of variation of the grid frequency, as following:

$$\Delta P_{el}^{pu} = k \cdot H_{pu} \cdot \frac{d\omega_{pu}}{dt}$$

Where:

- $k$  is a factor for synthetic inertia
- $H$  is the constant of inertia, expressing the kinetic energy stored in the rotor moving parts. By definition:

$$H = \frac{\text{kinetic energy stored in the rotor at synchronous speed}}{\text{nominal power}} = \frac{1}{2} \cdot \frac{I \cdot \omega_n^2}{P_n} \quad [\text{sec}]$$

As the base of  $H$  is by definition 1 [sec],  $H_{pu}$  is given by the following expression:

$$H_{pu} = \frac{1}{2} I_{pu} = \frac{1}{2} M_{pu}$$

- $I_{pu}$  is the wind turbine inertia in pu in the wind turbine base.

## References

- [1] Hansen, M.O.L. (2000). Aerodynamics of Wind Turbines: Rotors, Loads and Structure, James & James, London, U.K., 144p.
- [2] Akhmatov, V. (2007). Induction Generators for Wind Power, Multi-science Publishing Co., Essex, U.K., June 2007, ISBN 0 906522 40 4, Available on: [www.multi-science.co.uk](http://www.multi-science.co.uk).
- [3] DNV/Risø, Guidelines for design of wind turbines, 3rd Edition (2007).
- [4] Hansen, A.D., Generators and Power Electronics for Wind Turbines. Chapter in Wind Power in Power Systems, John Wiley & Sons, Ltd, 2005, 24 p.
- [5] Larsson, Å. (2000) The Power quality of Wind Turbines, Ph.D. report, Chalmers University of Technology, Göteborg, Sweden.
- [6] Hansen A.D., Hansen L.H., Market penetration of different wind turbine concepts over the years, EWEC 2007, Milano.

DTU Vindenergi er et institut under Danmarks Tekniske Universitet med en unik integration af forskning, uddannelse, innovation og offentlige/private konsulentopgaver inden for vindenergi. Vores aktiviteter bidrager til nye muligheder og teknologier inden for udnyttelse af vindenergi, både globalt og nationalt. Forskningen har fokus på specifikke tekniske og videnskabelige områder, der er centrale for udvikling, innovation og brug af vindenergi, og som danner grundlaget for højt kvalificerede uddannelser på universitetet.

Vi har mere end 240 ansatte og heraf er ca. 60 ph.d. studerende. Forskningen tager udgangspunkt i ni forskningsprogrammer, der er organiseret i tre hovedgrupper: vindenergisystemer, vindmølleteknologi og grundlag for vindenergi.

---

**Danmarks Tekniske Universitet**

DTU Vindenergi  
Nils Koppels Allé  
Bygning 403  
2800 Kgs. Lyngby  
Telefon 45 25 25 25

[info@vindenergi.dtu.dk](mailto:info@vindenergi.dtu.dk)  
[www.vindenergi.dtu.dk](http://www.vindenergi.dtu.dk)